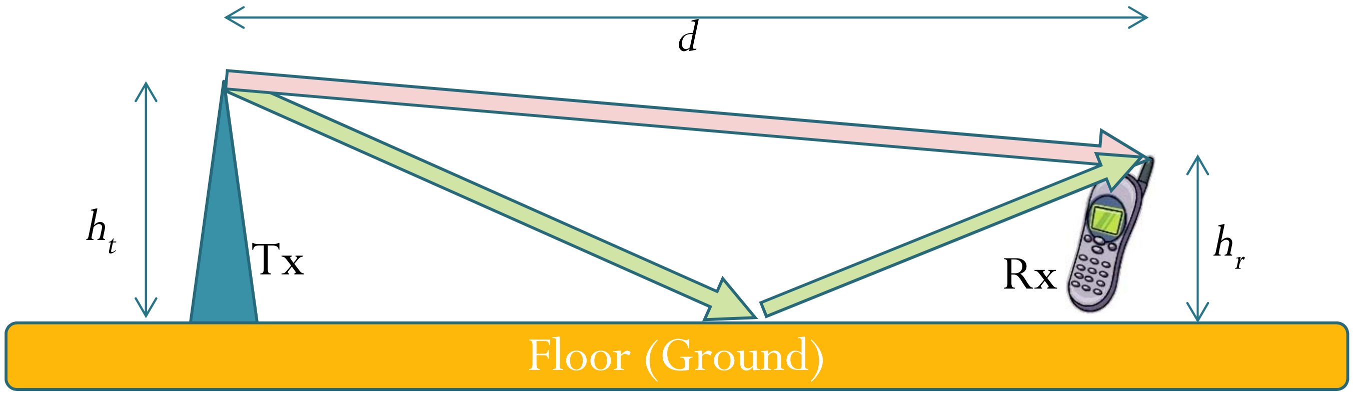
Ex. Two-Ray Model

$$\text{Delay spread} = \frac{r_2}{c} - \frac{r_1}{c}$$

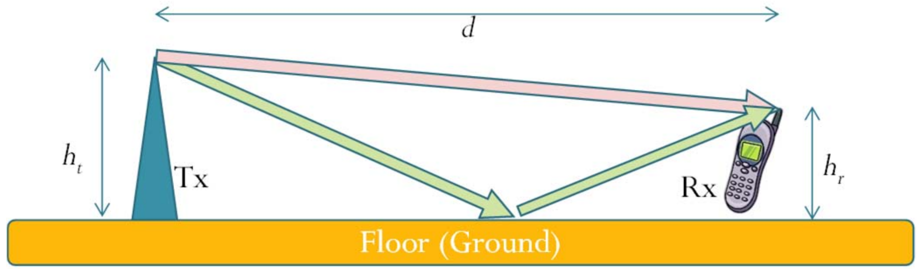
$$y(t) = \frac{\alpha}{r_1}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_2}{c}\right)\right)$$

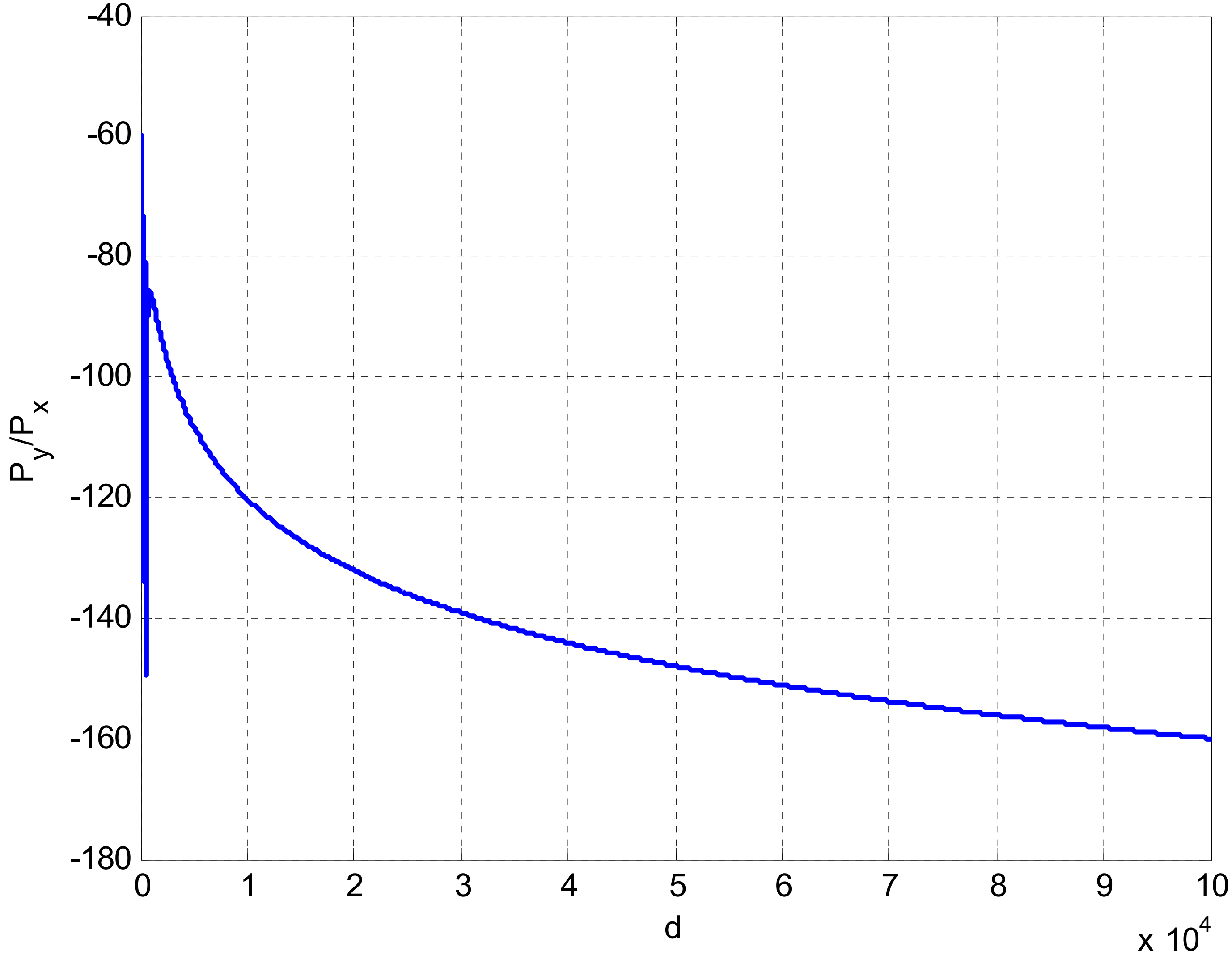
$$\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1}e^{-j2\pi f_c \frac{r_1}{c}} - \frac{\alpha}{r_2}e^{-j2\pi f_c \frac{r_2}{c}}\right|^2 = \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi f_c \frac{r_2 - r_1}{c}}\right|^2$$



Assume ground reflection coefficient = -1.

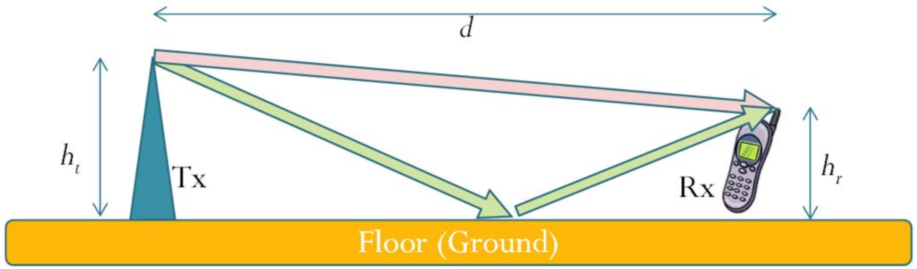
Ex. Two-Ray Model

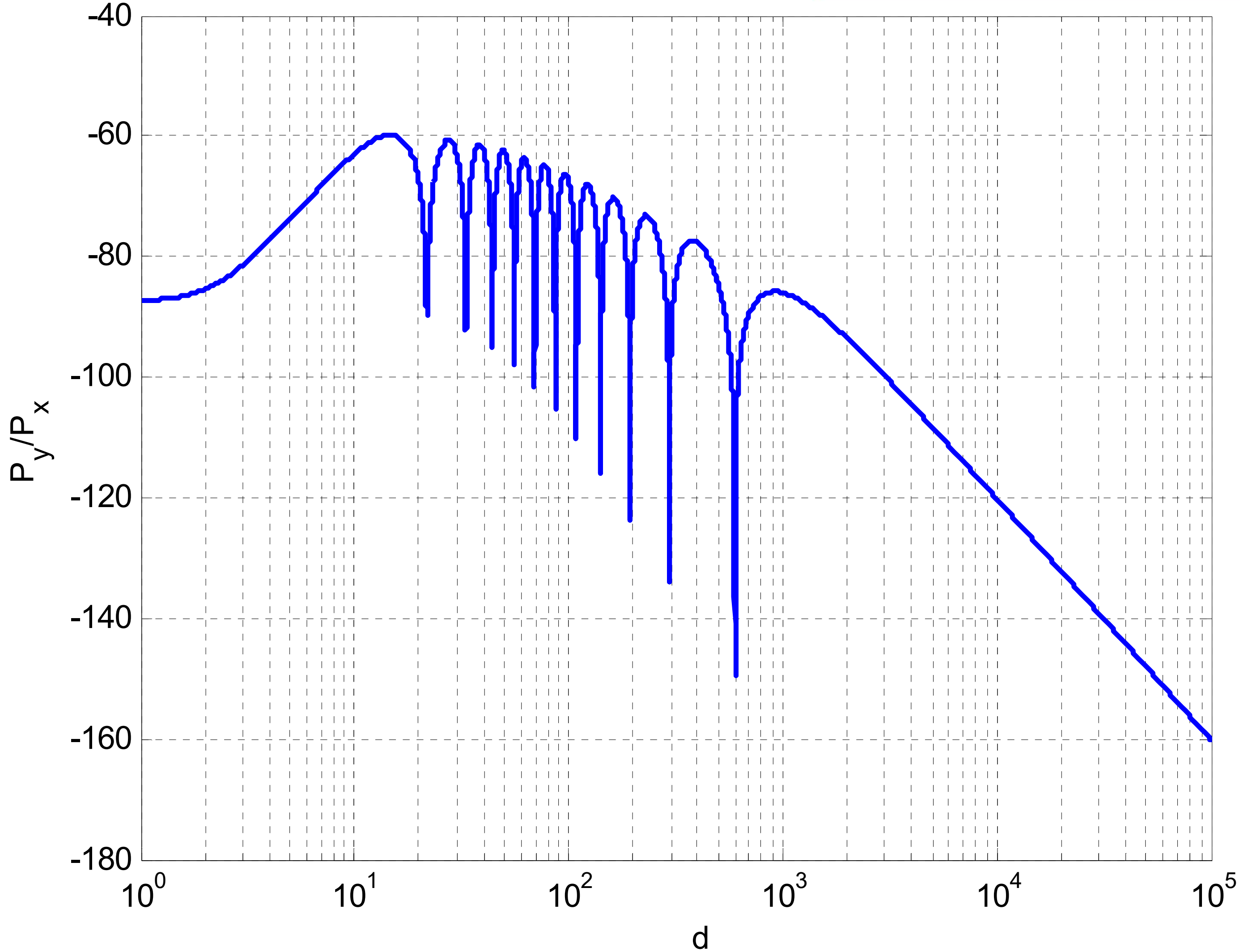




$$f = 900 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

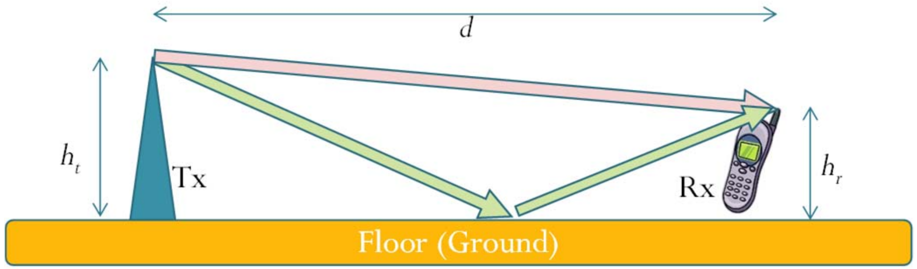
Ex. Two-Ray Model

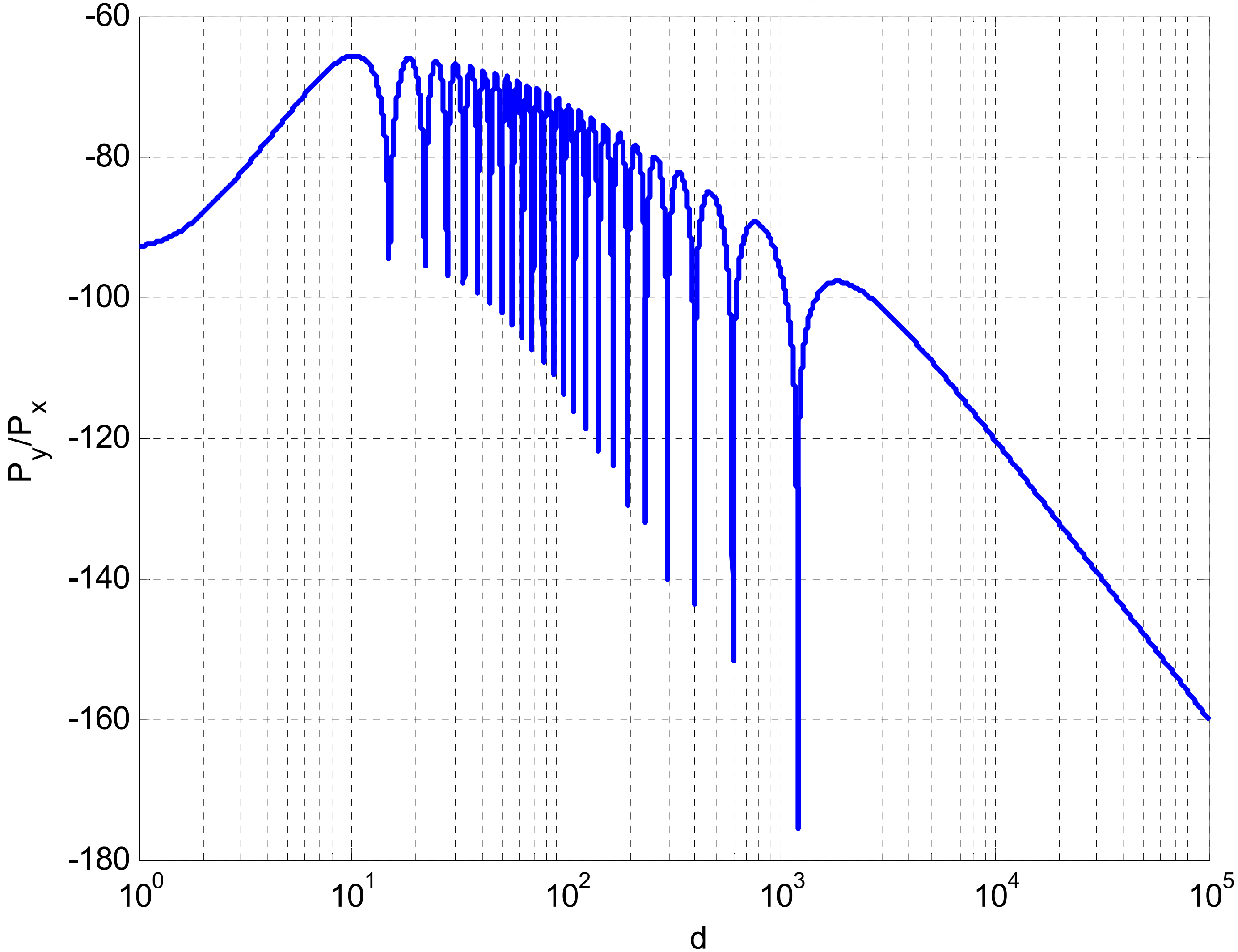




$$f = 900 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model





$$f = 1800 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

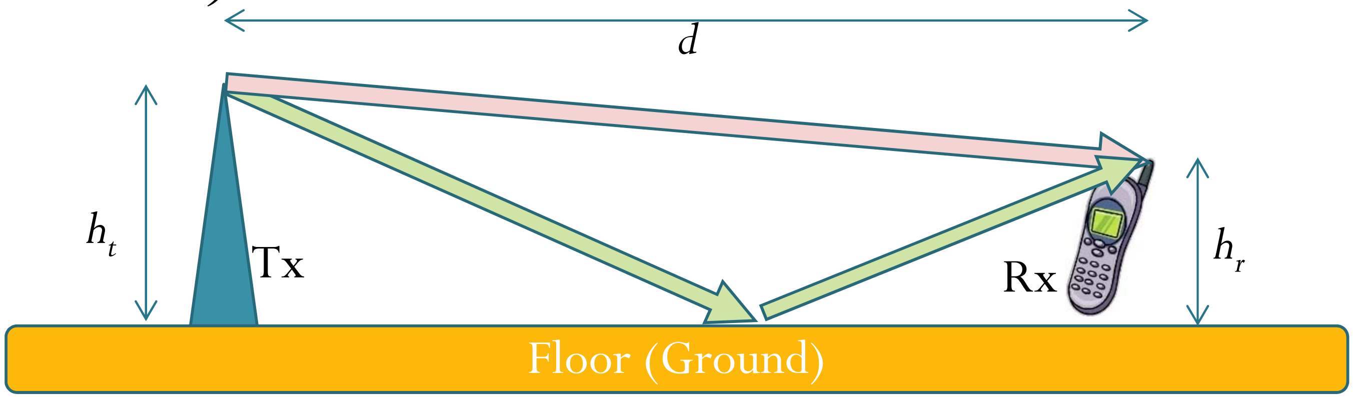
Ex. Two-Ray Model (Approximation)

$$d \gg h_t, h_r$$

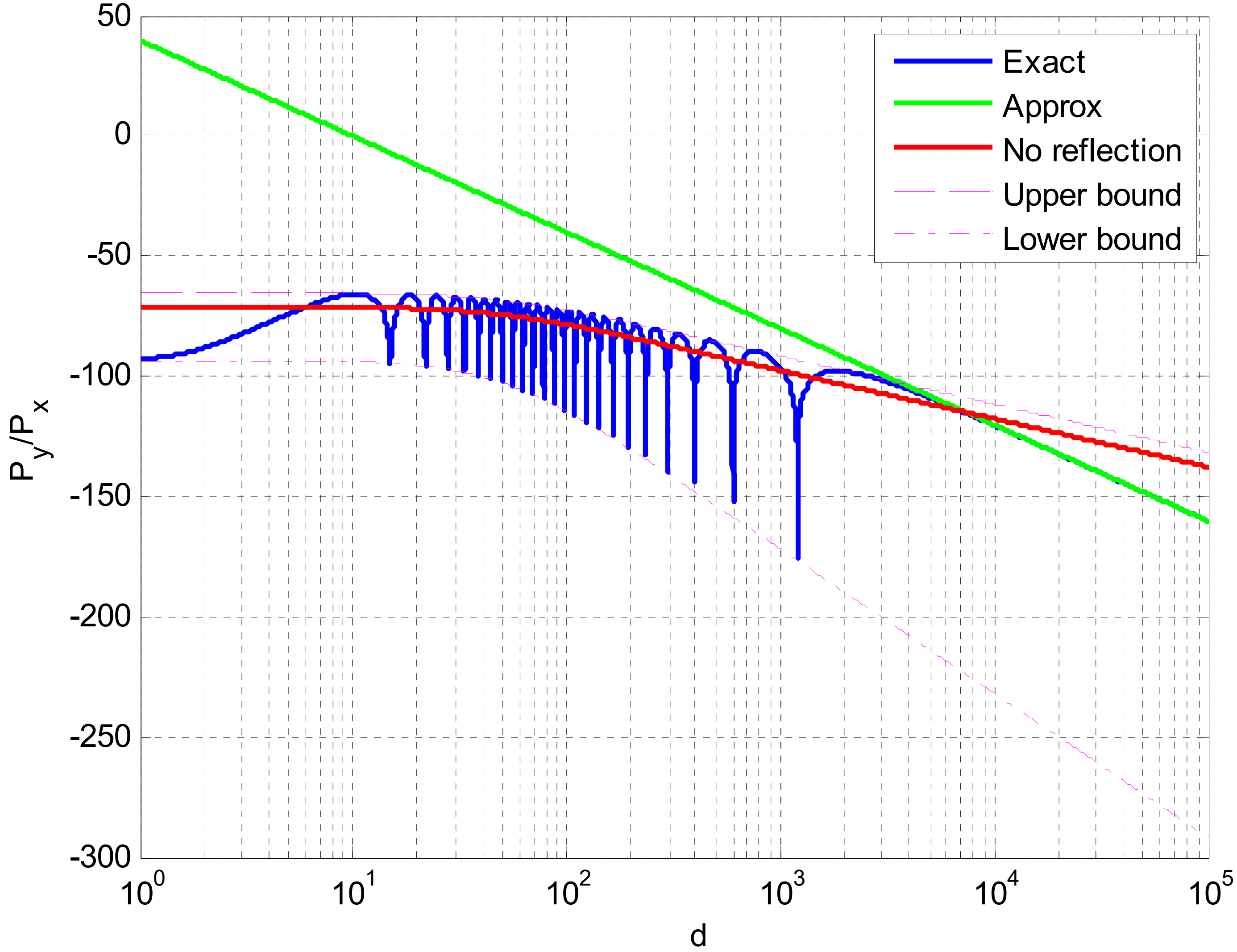
$$\frac{P_y}{P_x} \approx \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi\frac{2h_t h_r}{\lambda}} \right|^2 \approx \frac{\alpha}{d} \left| 1 - e^{-j2\pi\frac{2h_t h_r}{\lambda d}} \right|^2$$

$$\approx \left(\frac{\alpha}{d}\right)^2 \left| 1 - \left(1 - j2\pi\frac{2h_t h_r}{\lambda d}\right) \right|^2 = \frac{\alpha^2}{d^2} \left| j2\pi\frac{2h_t h_r}{\lambda d} \right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2}\right)^2$$

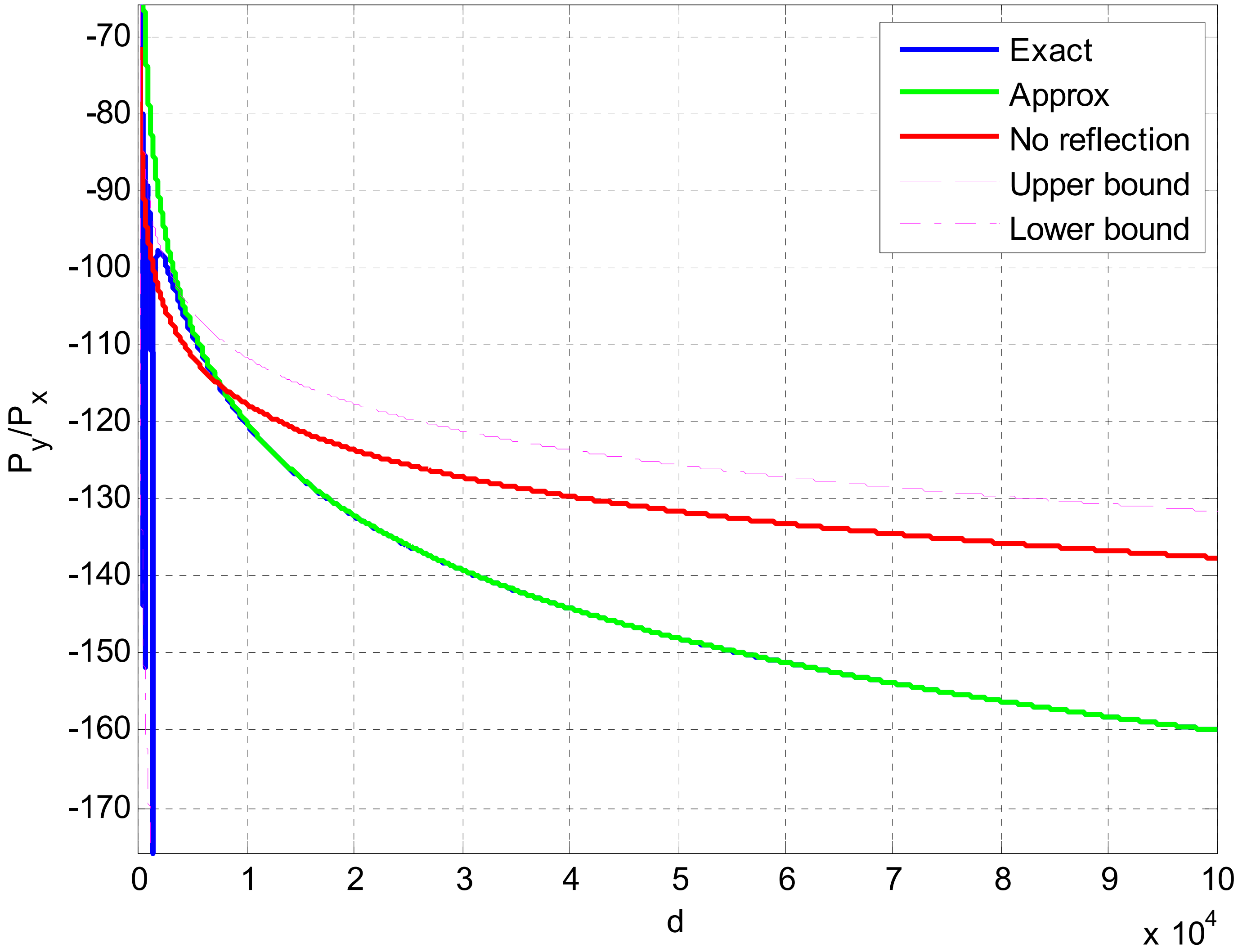
$$= \left(\frac{\sqrt{G_{Tx} G_{Rx}}\, h_t h_r}{d^2}\right)^2 \propto \frac{1}{d^4}$$



Ex. Two-Ray Model



Ex. Two-Ray Model



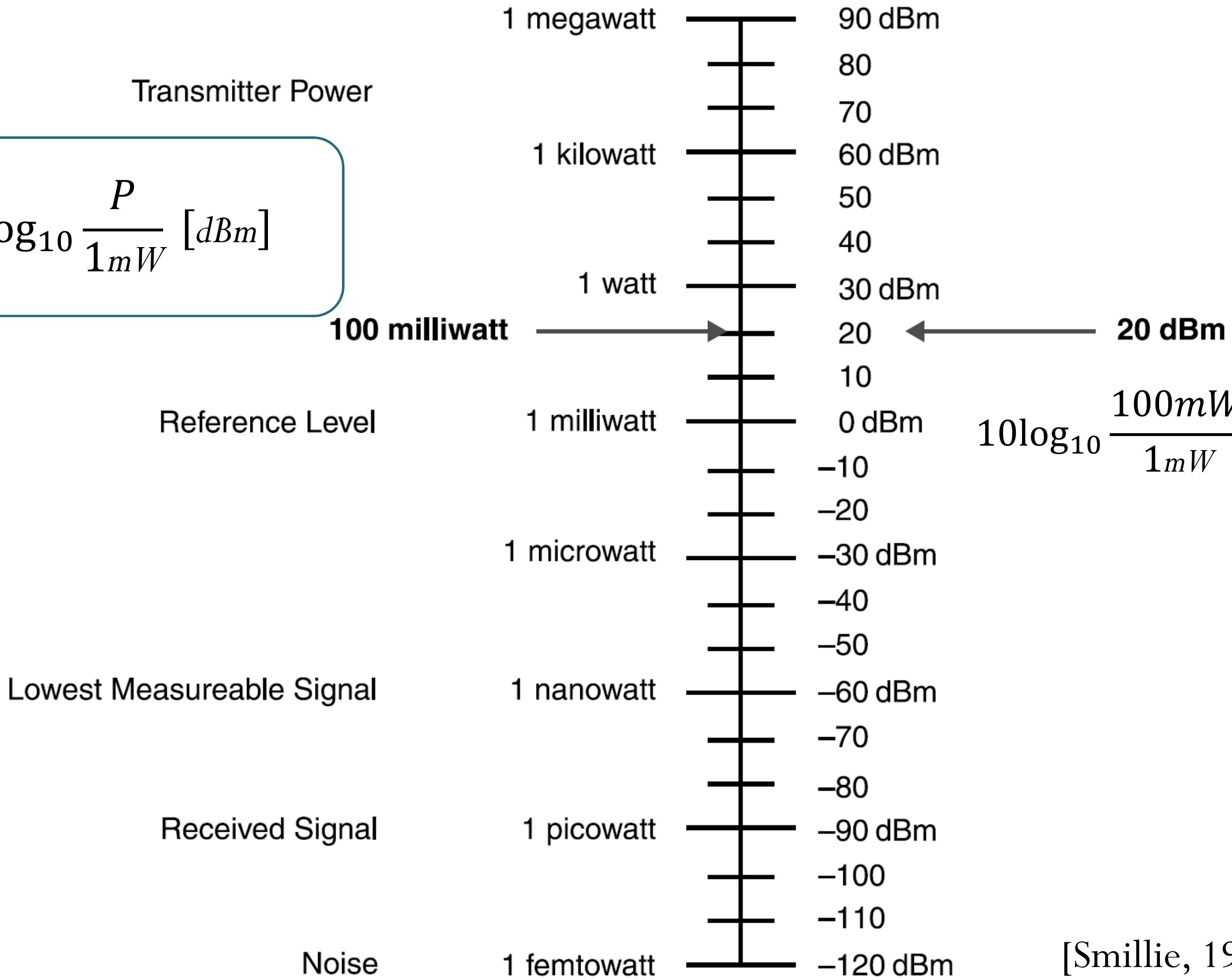
dBm

- The range of RF power that must be measured in cellular phones and wireless data transmission equipment varies from
  - hundreds of watts in base station transmitters to
  - picowatts in receivers.
- For calculations to be made, all powers must be expressed in the same power units, which is usually **milliwatts**.
  - A transmitter power of 100 W is therefore expressed as 100,000mW. A received power level of 1 pW is therefore expressed as 0.000000001mW.
- Making power calculations using decimal arithmetic is therefore complicated.
- To solve this problem, the dBm system is used.

[Scott and Frobenius, 2008, Fig 1.1]

Range of RF Power in Watts and dBm

$$P\ [W] = 10\log_{10}\frac{P}{1mW}\ [dBm]$$



$$10\log_{10}\frac{100mW}{1mW} = 20\ [dBm]$$

[Smillie, 1999, p 11]

dB and dBm

- The decibel scale expresses factors or ratios logarithmically.
- Unitless dB value
  - Represent power ratio: $10\log_{10}\frac{P_2}{P_1}$
- dB value with a unit
  - Represent the signal power itself:

$$P[\text{dBW}] = 10\log_{10}\frac{P}{1\text{ W}}, \qquad P[\text{dBm}] = 10\log_{10}\frac{P}{1\text{ mW}}$$

  - Note that $P[\text{dBm}] = P[\text{dBW}] + 30$

Remark

- Adding dB values corresponds to multiplying the underlying factors, which means multiplying the units if they are present.
- It is therefore appropriate to add unitless dB values to a dB value with a unit (such as dBm)
  - The result is still referred to that unit.
  - Ex: 17 dBm + 13 dB – 6 dB = 24 dBm
    - Correspond to 50 mW × 20 / 4 = 250 mW.

Doppler Shift: 1D Move

- At the transmitter, suppose we have

$$\sqrt{2P_t}\cos(2\pi f_c t + \phi)$$

- At distance r (far enough), we have

Time to travel a distance of r

$$\frac{\alpha}{r}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r}{c}\right) + \phi\right)$$

- If moving, r becomes $r(t)$.
- If moving ***away*** at a constant velocity v, then $r(t) = r_0 + vt$.

$$\frac{\alpha}{r(t)}\cos\left(2\pi f_c\left(t - \frac{r_0 + vt}{c}\right) + \phi\right) = \frac{\alpha}{r(t)}\cos\left(2\pi\left(f_c - f_c\frac{v}{c}\right)t - 2\pi f_c\frac{r_0}{c} + \phi\right)$$

Frequency shift

$$\Delta f = \frac{v}{\lambda}$$

Review: Instantaneous Frequency

For a generalized sinusoid signal

$$A\cos\left(\theta(t)\right),$$

the **instantaneous frequency** at time t is given by

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

When $\theta(t) = 2\pi f_c\left(t - \frac{r(t)}{c}\right) + \phi,$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) = f_c - \frac{f_c}{c}\frac{d}{dt}r(t) = f_c - \frac{1}{\lambda}\frac{d}{dt}r(t)$$

Frequency shift

Big Picture

Transmission impairments in cellular systems

Physics of radio propagation	Attenuation (Path Loss) Shadowing Doppler shift Inter-symbol interference (ISI) Flat fading Frequency-selective fading
Extraneous signals	Co-channel interference Adjacent channel interference Impulse noise White noise
Transmitting and receiving equipment	White noise Nonlinear distortion Frequency and phase offset Timing errors

[Myung and Goodman, 2008, Table 2.1]